



## Propuesta de Trabajo Fin de Grado del Doble Grado en Física y Matemáticas

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*Área de conocimiento:* Física y Información Cuántica

*Responsable de cotutorización:*

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*Área de conocimiento:*

*(Rellenar sólo en caso de que la propuesta esté realizada a través de un estudiante)*

*Estudiante que propone el trabajo:*

*Título:* Super Replication of Quantum Channels

*Número de créditos:* 12 ECTS

Tipología del trabajo (marcar una o varias de las siguientes casillas):

1. Revisiones y/o trabajos bibliográficos sobre el estado actual de aspectos específicos relacionados con la titulación
2. Estudio de casos, teóricos o prácticos, relacionados con la temática de la titulación, a partir del material disponible en los centros  X
3. Trabajos experimentales, de toma de datos de campo, de laboratorio, etc.
4. Elaboración de nuevas prácticas de laboratorio
5. Elaboración de un informe o un proyecto en el ámbito del grado de naturaleza profesional
6. Trabajos relacionados con las prácticas externas

### *Descripción y resumen de contenidos:*

Quantum information cannot be copied [1]: this statement is true if one demands *exact* replicas in a deterministic fashion. Probabilistic cloning machines—where exact replicas are produced with some finite, non-zero probability, do exist. On the opposite extreme some deterministic replication machines have also been constructed, whereby approximate replicas of the required state are produced.

One such phenomenon is *super-replication* [2,3]. Here we require a machine that, given  $N$  copies of a quantum state known to belong in some restricted set, the machine outputs  $M > N$  approximate replicas such that the error in the approximation becomes vanishingly small as  $N$  becomes very large. It was shown that the most replicas one can possibly obtain from such a machine is  $N^2$  [4] (hence the name super-replication), and that such a machine must necessarily be probabilistic [2].

However, if one tries to super-replicate quantum gates, then it was shown that super-replication can be *deterministic* [5,6]. Nothing, however, is known about the most general quantum operations possible. The question is of fundamental importance in quantum information theory, communication as well as computation. An answer to this question also leads to efficient (and possibly universal) compression algorithms for quantum operations (of vital importance to quantum simulation) as well as algorithms for interconversion between quantum information resources.

### *Actividades a desarrollar:*

The aim of this project is to investigate whether it is possible to super-replicate experimentally relevant quantum operations—pertaining to noisy dynamical evolutions described by Lindblad master equations. Specifically, we will look at all those quantum operations that respect rotational symmetry, i.e., they conserve the total angular momentum. To do so you will require to learn and apply the following tools

1. The formalism of quantum operations (Completely Positive Trace Preserving maps, and their associated representations)
2. Group representation theory (invariant subspaces, Schur-Weyl duality, Spherical Tensors and the Wigner-Eckart theorem).

### *Objetivos planteados*

Characterize all  $SU(2)$  invariant quantum operations in terms of their irreducible spherical tensors

Apply quantum resource theoretic techniques to investigate the extend to which such operations can be super replicated

Derive upper and lower bounds on the super-replication of such general quantum operations

Investigate several protocols inspired by previous works on super replication of quantum states and quantum gates.

## Bibliografía

- [1] Wootters, W. K., Zurek, W. H. A single quantum cannot be cloned. [Nature, 299, 802-803.](#)
- [2] Chiribella, G., Yang, Y. and Yao, A.C.C. Quantum replication at the Heisenberg limit. [Nature Communications, 4, 1.](#)
- [3] Chiribella, G., Yang, Y. Quantum superreplication of states and gates. [Frontiers of Physics, 11, 1-19.](#)
- [4] Sekatski, P., Skotiniotis, M., Dür, W. No-signaling bounds for quantum cloning and metrology. [PRA, 92, 022355.](#)
- [5] Dür, W., Sekatski, P., Skotiniotis, M. Deterministic superreplication of one-parameter unitary transformations. [PRL, 114, 120503.](#)
- [6] Chiribella, G., Yang, Y., & Huang, C. Universal superreplication of unitary gates. [PRL, 114, 120504.](#)

Firma del estudiante  
(solo para trabajos propuestos por estudiantes)

Firma del responsable de tutorización  
(solo para trabajos propuestos por estudiantes)

Firma del responsable de cotutorización (*en su caso*)  
(solo para trabajos propuestos por estudiantes)

En Granada, a 24 de Abril de 2023.